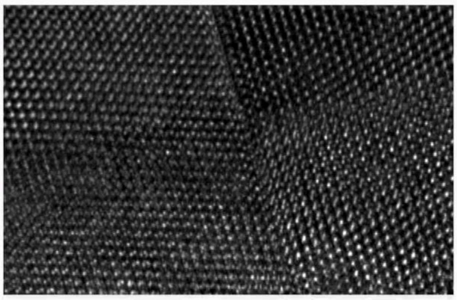
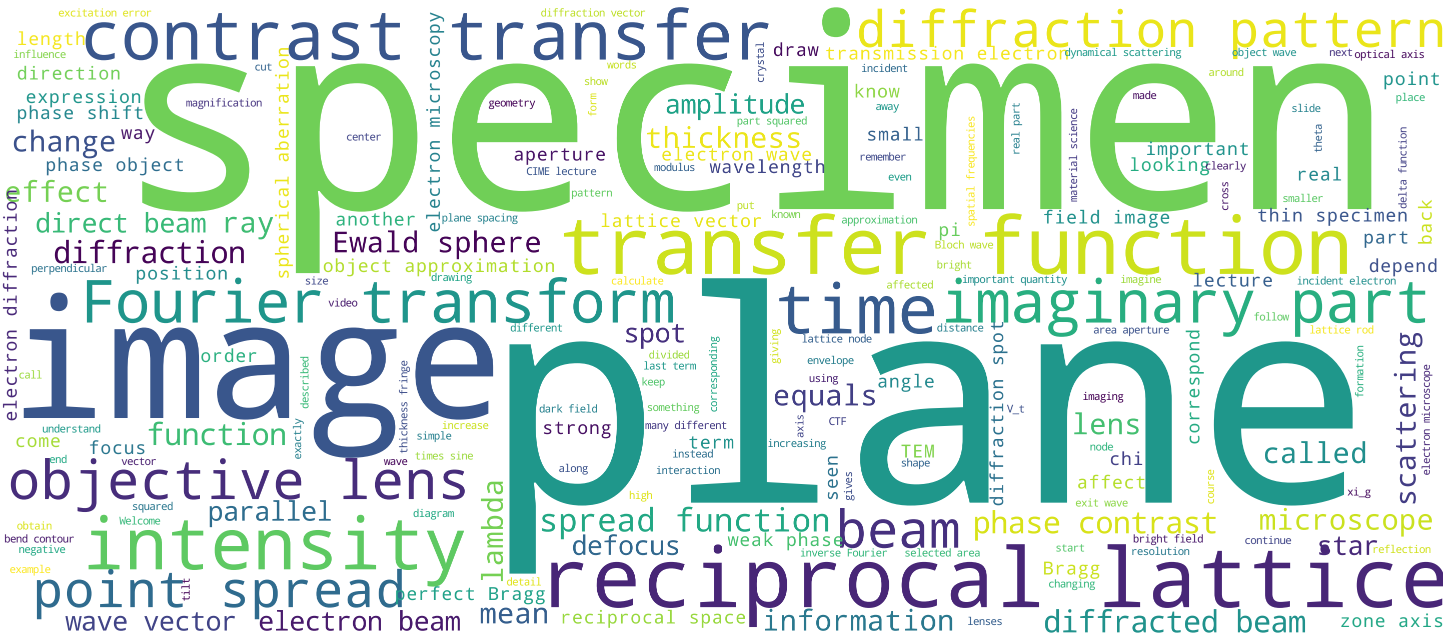
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE



Phase contrast: (weak) phase objects

Transmission Electron Microscopy

Prof. C. Hébert & Dr. D. Alexander



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Video

EPFL

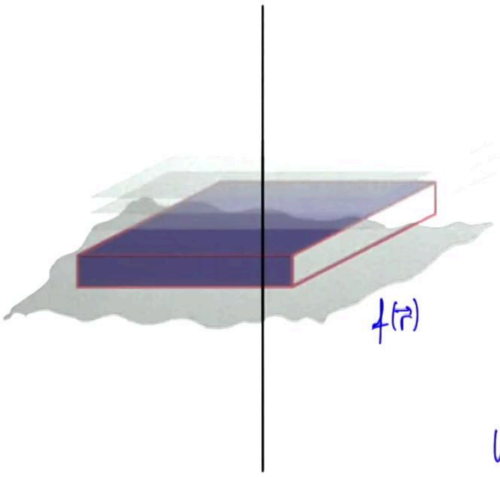
Welcome to CIME's lecture on transmission electron microscopy for material science. In this video, we will see the effect of a thin specimen which only affects the phase, the weak phase object approximation and what the influence is when we consider the contrast transfer function.

Notes

Summary

0m 05s

The specimen



$f(\vec{r}) = A(\vec{r})\, e^{-i\phi_t(\vec{r})}$

Thin specimen $\Rightarrow$ only the projected potential $V_t(\vec{r})$ affects the phase

$d\phi = \frac{\pi}{\lambda E} V_t(\vec{r})$

Very thin specimen $A(\vec{r}) = 1$

$\frac{\pi}{\lambda E} V_t(\vec{r}) \ll 1 \Rightarrow f(\vec{r}) = 1 - \frac{i\pi}{\lambda E} V_t(\vec{r})$

Weak Phase Object Approximation WPOA

Transmission Electron Microscopy

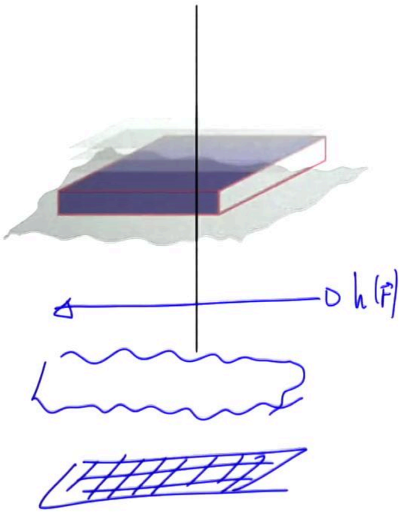
So now I am back with my specimen, the electron wave that comes on the specimen and is a plain wave. And I have the electron wave after the specimen called f(r). This is affected both in amplitude and phase. At that position I have the phase shift, which depends on the thickness of the specimen t. Here is the amplitude and the phase. If I have a thin specimen, only the projected potential will affect the phase. I can write the phase shift as pi over lambda E V_t(r). Since I have a thin specimen, the phase shift is small. Lambda is the wavelength of the electrons and E the electron energy. Now if I have a very thin specimen, the amplitude is not affected, so A(r) = 1. And the phase shift is extremely small, so pi over lambda E V_t(r) is much smaller than 1. With this I can rewrite f(r). This is called the “weak phase object approximation”. We will now look at what is the influence of the objective lens on this object wave.

Notes

Summary

0m 25s

The Weak Phase Object Approximation



$$f(\vec{r}) = 1 - \frac{i\pi}{\lambda E} V_t(\vec{r})$$

$H(\vec{u}) = A(\vec{u})\, B(\vec{u})\, E(\vec{u}) \quad \text{FT} \quad h(\vec{r})$

after objective lens $g(\vec{r}) = f(\vec{r}) \otimes h(\vec{r})$

Measured quantity $I(\vec{r}) = |g|^2 = g \cdot g^*$

$$I(\vec{r}) = \left| \left(1 - \frac{i\pi}{\lambda E} V_t(\vec{r})\right) \otimes h(\vec{r}) \right|^2$$

$$= \left| (1 \otimes h(\vec{r})) - \frac{i\pi}{\lambda E} V_t(\vec{r}) \otimes (\mathrm{Re}(h) + i\, \mathrm{Im}(h)) \right|^2$$

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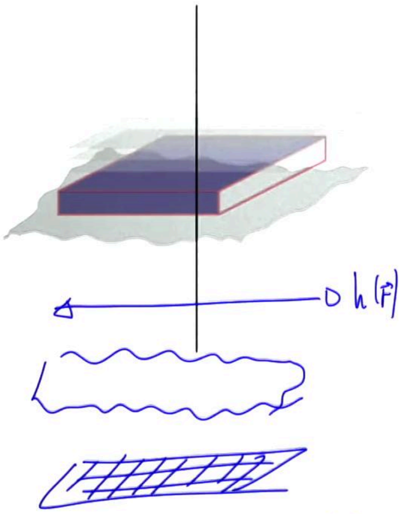
So we keep the specimen object wave in the weak phase object approximation and we now look at the objective lens. The objective lens is described by the point spread function and its Fourier transform, the contrast transfer function, which is the Fourier transform of the point spread function. After the objective lens, we have the electron wave formed as an image, which is the convolution of the exit wave and the point spread function. But what we measure is then the intensity projected on a recording device. So the measured quantity I(r) has the following expression: g times its complex conjugate. I will calculate this using the point spread function and the approximation for the exit wave. I now insert this expression for f, but I keep the point spread function. I have a real part and an imaginary part. So now I will consider the two real and imaginary parts of the point spread function. So I will first expand the convolution product and see what the first term is doing. And then for the second part, I will need to consider separately real and imaginary part of h. I will first take care of this first quantity, 1 convoluted with h. If I want to get this, I can consider its Fourier transform.

Notes

Summary

2m 20s

The Weak Phase Object Approximation



$$f(\vec{r}) = 1 - \frac{i\pi}{\lambda E} V_t(\vec{r})$$

$H(\vec{u}) = A(\vec{u})\, B(\vec{u})\, E(\vec{u}) \quad FT \quad h(\vec{r})$

after objective lens $g(\vec{r}) = f(\vec{r}) \otimes h(\vec{r})$

Measured quantity $I(\vec{r}) = |g|^2 = g \cdot g^*$

$$I(\vec{r}) = \left| \left(1 - \frac{i\pi}{\lambda E} V_t(\vec{r})\right) \otimes h(\vec{r}) \right|^2$$

$$= \left| \underbrace{\left(1 \otimes h(\vec{r})\right)}_{A=1} - \frac{i\pi}{\lambda E} V_t(\vec{r}) \otimes \left(\mathrm{Re}(h) + i\, \mathrm{Im}(h)\right) \right|^2$$

$A = FT^{-1}\left(\delta(\vec{u}) \cdot H(\vec{u})\right)$
$= 1$

Transmission Electron Microscopy

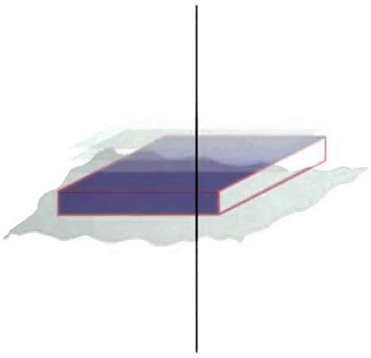
So this is the inverse Fourier transform of its Fourier transform, which will transform this into a multiplicative product. The Fourier transform of 1 is a delta function, I'm now entering reciprocal space. So I have u times the Fourier transform of the point spread function is the contrast transfer function. This is A and A is equal to that. Okay, this just picks the value of H for u equals 0, which is 1, so it is also a delta function. The inverse Fourier transform of a delta function is unity. So A is equal to 1. So we are reporting now this equation on the next slide.

Notes

Summary

4m 27s

The Weak Phase Object Approximation



$$f(\vec{r}) = 1 - \frac{i\pi}{\lambda E} V_t(\vec{r})$$

$$I(\vec{r}) = \left| 1 - \frac{i\pi}{\lambda E} V_t(\vec{r}) \otimes (\Re(h) + i\Im(h)) \right|^2$$

$$= \left| 1 + \frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Im(h) + \frac{i\pi}{\lambda E} V_t(\vec{r}) \otimes \Re(h) \right|^2$$

$$I(\vec{r}) = \left(1 + \frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Im(h)\right)^2 + \left[\frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Re(h)\right]^2$$

$$= \underbrace{1 + 2\frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Im(h)} + \cancel{\left(\frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Im(h)\right)^2 + \left(\frac{\pi}{\lambda E} V_t(\vec{r}) \otimes \Re(h)\right)^2}$$

negligible

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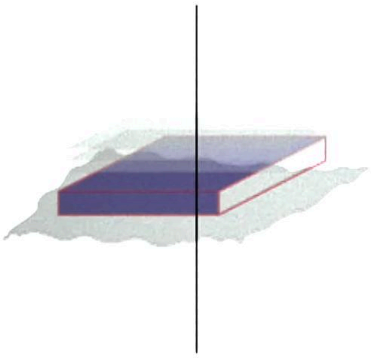
I will continue by expanding this term to separate real and imaginary parts. The real part will come from the multiplication of this imaginary part with this one, and the imaginary part from the last term. It is time now to calculate the modulus to get y. So the real part squared plus the imaginary part squared. Let us now come back to the weak phase object approximation. We build it by saying that this quantity is small with respect to 1. Now I will have this quantity, small with respect to 1 as well. And if you look well, this will be of order 2. And this can be simplified by keeping only the first order. The two last terms, are negligible. So the projected intensity only depends on the imaginary part of the point spread function.

Notes

Summary

5m 29s

The Weak Phase Object Approximation



$$f(\vec{r}) = 1 - \frac{i\pi}{\lambda E} V_t(\vec{r})$$

$$I(\vec{r}) = 1 + \frac{2\pi}{\lambda E} V_t(\vec{r}) \otimes \Im(\mathrm{h}(\vec{r}))$$

$CTF = A.E.e^{i\chi}$ $\quad$ $\mathrm{Im}(CTF) = A.E \sin \chi(\vec{u})$

$T(\vec{u}) = A(\vec{u})\, E(\vec{u})\, 2 \sin \chi(\vec{u})$

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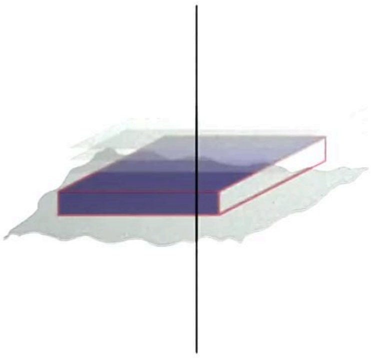
Since now the only important quantity is the imaginary part of the point spread function, that also means that for the contrast transfer function, I will only need to consider its imaginary part. The CTF was A times the envelope times B, which is exponential i chi. If I take the imaginary part of the CTF, I end up with A times E, both are real, times sine of chi(u). Now I also have this factor of two which arised due to the approximation we made before. So I can define the phase contrast transfer function T(u) as being A(u), E(u) times 2 sine chi(u). So this phase contrast transfer function is a characteristic of the microscope. It includes the effect of apertures, the envelopes and the aberrations of the objective lens. It will act as a filter in reciprocal space and govern the transfer of information as a function of spatial frequencies.

Notes

Summary

7m 06s

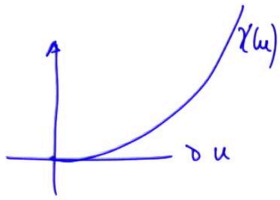
Phase Contrast Transfer Function

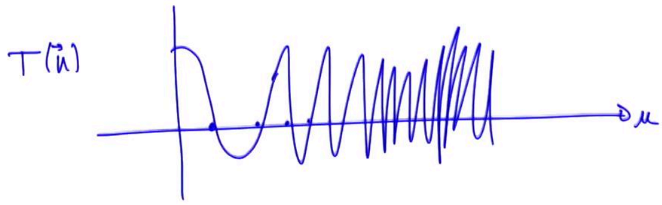


$$T(\vec{u}) = A(\vec{u})E(\vec{u})2\sin\chi(\vec{u})$$

- If we only have defocus and spherical aberration:

$$\chi(\vec{u}) = \pi\Delta f\lambda u^2 + \frac{1}{2}\pi C_s\lambda^3 u^4$$





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So we have to remember now this very important quantity called the phase contrast transfer function. If we only have defocus and spherical aberration, we have an expression for chi(u). This is this expression depending on the defocus delta f and the spherical aberration coefficient C_s. So now, we have the fast growing function as a function of u, and we take the sine of this. You can imagine that we have chi(u), which will grow for increasing u very fast. If you take the sine of this, you can expect the transfer function T(u) to have a very complicated form with something I don't know how it will look like at the beginning. But for sure when I increase, go to increasing u, I will get faster and faster oscillations. What does it mean to have such a strange function? If it crosses the horizontal axis, I have to transfer function equal to 0. That means, I will not transfer any information. If I have a positive or negative, I will have inversion of contrast. This is the spatial frequency u. So that as a function of u, I will have very complicated different transfer of information.

Notes

Summary

8m 49s

So now, you have seen how we come to the phase contrast transfer function, which is very important to explain the transfer of information in the case of the weak phase object approximation. In the next video, we will look in more details at the phase contrast transfer function as a function of the microscope capabilities, C_s and defocus, and see how it affects the image that we can record.

Notes

Summary

10m 53s